Algebra II Texas Mathematics: Unpacked Content

What is the purpose of this document?
To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the new math standards.

What is in the document?
Descriptions of what each standard means a student will know, understand, and be able to do. The “unpacking” of the standards done in this document is an effort to answer a simple question “What does this standard mean that a student must know and be able to do?” and to ensure the description is helpful, specific and comprehensive for educators.
At A Glance:

New to Algebra II:
- Write equations of parabolas using specific attributes.
- Specifically solve systems using three linear equations and systems consisting of two equations, one linear and one quadratic.
- Use Gaussian elimination method of solving systems of equations.
- Addition of the cubic and cube root functions
- Composition of two functions
- Identify extraneous solutions of square root equations
- Formulate absolute value of linear equations
- Solve absolute value linear equations and inequalities
- Transformations with cubic, cube root and absolute value functions
- Addition, subtract, and multiply polynomials
- Determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two
- Determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and of degree two
- Rewrite radical expressions that contain variables to equivalent forms
- Solve equations involving rational exponents
- Use linear, quadratic, and exponential functions to model data
- Use regression methods to write linear, quadratic, and exponential models
- TEKS language no longer refers to the different types of functions as “parent” functions, they are simply referred to as “functions”

Algebra II TEKS moved to Algebra I
- Writing equations that are parallel and perpendicular to the x- and y- axis (zero and undefined slope)
- Dividing polynomials
- Solving and graphing quadratics
- Make connections between standard and vertex form in a quadratic function
- Writing quadratic functions specifically using the vertex and another point
- Predict the effects of changes in a, h, and k using the graph of y=a(x-h)^2+k
- Predict the effects of changes in a and b using the form f(x)=ab^x
- Find the domain and range of exponential functions
- Write exponential functions

Algebra II TEKS moved to Pre-Calculus
- Graph and write an equation of an ellipse and a hyperbola
- Solving rational inequalities

Algebra II TEKS moved to Geometry
- Graph and write an equation of a circle
Instructional Implications for 2015-16:

- Some teachers have already been teaching many of the new Algebra II TEKS, important to cover in depth now
- Remember to cover the gaps created by TEKS that have moved up or down

Professional Learning Implications for 2015-16:

- Important for all to know the changes in TEKS and how that affects the students’ math progress
- Collaboration among Algebra I, Algebra II, Pre-Calculus, and Geometry teachers to ensure no gaps occur
Algebra II Primary Focal Areas:

The Primary Focal Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

1. The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

2. The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

3. In Algebra II, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. Students will broaden their knowledge of quadratic functions, exponential functions, and systems of equations. Students will study logarithmic, square root, cubic, cube root, absolute value, rational functions, and their related equations. Students will connect functions to their inverses and associated equations and solutions in both mathematical and real-world situations. In addition, students will extend their knowledge of data analysis and numeric and algebraic methods.

4. Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.
<table>
<thead>
<tr>
<th><strong>Mathematical Process Standards</strong></th>
<th><strong>Unpacking</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:</td>
<td><strong>What does this standard mean that a student will know and be able to do?</strong></td>
</tr>
<tr>
<td><strong>1(A)</strong> apply mathematics to problems arising in everyday life, society, and the workplace;</td>
<td><strong>Apply</strong></td>
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<table>
<thead>
<tr>
<th><strong>Mathematical Practices</strong></th>
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<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
</tr>
<tr>
<td>6. Attend to precision.</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
</tr>
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<td>8. Look for and express regularity in repeated reasoning.</td>
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<td><strong>1(B)</strong> use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;</td>
<td><strong>Use, Formulate, Determine, Justify, Evaluate</strong></td>
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<td><strong>1(C)</strong> select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;</td>
<td>Select, Solve</td>
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<td><strong>1(D)</strong> communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;</td>
<td><strong>Communicate:</strong> Have the students: &lt;br&gt;1) Keep a journal &lt;br&gt;2) Participate in online forums for the classroom like Schoology or Edmoto. &lt;br&gt;3) Write a summary of their daily notes as part of the assignments.</td>
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<td><strong>1(E)</strong> create and use representations to organize, record, and communicate mathematical ideas;</td>
<td><strong>Create, Use</strong> The students can: &lt;br&gt;1) Create and use graphic organizers for things such as parent functions. This organizer can have a column for a) the name of the functions, b) the parent function equation, c) the graph, d) the domain, e) the range, f) any asymptotes. This organizer can be filled out all at once, or as you go through the lessons. They could even use this on assessments at the teacher’s discretion.</td>
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<td>1(F) analyze mathematical relationships to connect and communicate mathematical ideas; and</td>
<td>Analyze</td>
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<td>The student applies mathematical processes to understand that functions have distinct key attributes and understand the relationship between a function and its inverse. The student is expected to:</td>
<td>What does this standard mean that a student will know and be able to do?</td>
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<tr>
<td>2(A) graph the functions ( f(x) = \sqrt{x}, f(x) = \frac{1}{x}, ) ( f(x) = x^3, f(x) = 3\sqrt{x}, f(x) = x^b, f(x) =</td>
<td>x</td>
</tr>
</tbody>
</table>

* This could work here as well as in the example above. The graphs would be stressed at this standard.

**The students can:**
1) Create and use graphic organizers for things such as parent functions. This organizer can have a column for a) the name of the functions, b) the parent function equation, c) the graph, d) the domain, e) the range, f) any asymptotes. This organizer can be filled out all at once, or
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2(B) graph and write the inverse of a function using notation such as $f^{-1}(x)$; Graph, Write

The students should be given several graphs that may or may not be “functions”. In teams, the students will: a) identify points on the given graph, b) identify the domain and range of the given graphs, c) switch the x and y values, d) **graph** the results using the “new” points, e) **write** the domain and range of the result. The teacher can guide them to the idea that the **inverse** is the original graph reflected about the line $y = x$.  

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2(C) describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range; and Describe, Analyze

*This lesson will be similar to the one above, however this time the original functions should be specific functions that will demonstrate the relationships between the
inverse functions, i.e., quadratic and square root, logarithmic and exponential.

The students should be given several graphs that have inverses. In teams, the students will: a) identify points on the given graph, b) identify the domain and range of the given graphs, c) switch the x and y values, d) graph the results using the “new” points, e) write the domain and range of the result. The teacher can guide them to the idea that the inverse is the original graph reflected about the line y = x.

Next, the students should identify the type of function the resulting graph is, write its equation, identify its domain and range, and analyze any restrictions on the range.

Finally, have the students describe the relationship between a function and its inverse. This can be done as an exit ticket, a summary of the day’s work in a notebook, in a classroom online forum, or as a warmup for the next day’s work.

Describe the relationship between a function and its inverse for quadratic, square roots, logarithms, and exponentials.

Example:
Given the function \( f(x) = x^2 + 2x \), a table of values of points would include:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

A table of values for the inverse, \( f^{-1} \), would reverse the values for x and y.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
</table>
The graphs will be reflected across the identity function.

<p>| | |</p>
<table>
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<th></th>
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</tr>
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<td>0</td>
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<td>3</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
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</table>

Linear Functions, Equations, and Inequalities
The student applies mathematical processes to understand that functions have distinct key attributes and understand the relationship between a function and its inverse. The student is expected to:

2(D) use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.

Unpacking
What does this standard mean that a student will know and be able to do?

Use, Determine
Remind the students of composition of functions from Algebra 1. Explain what composition of functions is and how to do the work. Then give them several examples grouped into two groups: 1) These functions are inverses of each other, and 2) these functions are not inverses of each other. Ask them to determine what the inverses have in common that the non-inverse functions do not have in common. Lead them to the understanding that if two functions are inverses of each other, then f(g(x)) = x and g(f(x)) = x. If there is any other result, than the functions are not inverses of each other.

Systems of Equations and Inequalities
The student applies mathematical processes to formulate systems of equations and inequalities, use a variety of methods to solve

Unpacking
What does this standard mean that a student will know and be able to do?
and analyze reasonableness of solutions. The student is expected to:

| 3(A) formulate systems of equations, including systems consisting of three linear equations in three variables and systems consisting of two equations, the first linear and the second quadratic; | Formulate Help the students analyze application problems with 2 or 3 variables. They should practice formulating systems of equations from the given real-world problems. In this item, they are only formulating the systems, not solving them. Three examples are:

**Two linear equations:**
1) The admission fee at a small fair is $1.50 for children and $4.00 for adults. On a certain day, 2200 people enter the fair and $5050 is collected. How many children and how many adults attended?

**Three linear equations:**
2) Billy’s Restaurant ordered 200 flowers for Mother’s Day. They ordered carnations at $1.50 each, roses at $5.75 each, and daisies at $2.60 each. They ordered mostly carnations, and 20 fewer roses than daisies. The total order came to $589.50. How many of each type of flower was ordered?

**1 Linear and 1 Quadratic**
3) An acorn, falling from the top of a 45 foot tree is modeled by the equation \( h = -16t^2 + 45 \). Before it can hit the ground a squirrel hiding on a lower branch jumps out and intercepts it. If the squirrel’s movement is modeled by the equation \( h = -3t + 32 \), at what height did the squirrel intercept the acorn? |
3(B) solve systems of three linear equations in three variables by using Gaussian elimination, technology with matrices, and substitution;

<table>
<thead>
<tr>
<th>Solve</th>
</tr>
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</table>
| 1) This website has a good explanation of How to use Gaussian Elimination for a 3-variable system of equations: [http://mathworld.wolfram.com/GaussianElimination.html](http://mathworld.wolfram.com/GaussianElimination.html)  
2) Use inverse matrices to solve a 3-variable system of equations with the calculator. This should be done as part of the matrices unit.  
3) Solving by substitution is the same with 3-variable systems of equations as it is with 2-variable systems, there is just an extra step. |

What is the x-value of the solution to the system of equations below?

\[
\begin{align*}
x + y + z &= 8 \\
x + 2y &= 6 \\
y + z &= 4 
\end{align*}
\]

---

**Systems of Equations and Inequalities**
The student applies mathematical processes to formulate systems of equations and inequalities, use a variety of methods to solve and analyze reasonableness of solutions. The student is expected to:

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</table>

3(C) solve, algebraically, systems of two equations in two variables consisting of a linear equation and a quadratic equation;

<table>
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<th>Solve</th>
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<tbody>
<tr>
<td>Here is the problem from above with a good explanation of the answer:</td>
</tr>
</tbody>
</table>

**Question:**
An acorn, falling from the top of a 45 foot tree is modeled by the equation \( h = -16t^2 + 45 \). Before it can hit the ground a squirrel hiding on a lower branch jumps out and intercepts it. If the squirrel's movement is modeled by the equation \( h = -3t + 32 \), at what height did the squirrel intercept the acorn?
**Answer Explanation:**
You want to know when the height of the squirrel and the height of the acorn are the same, so take your two equations and make them equal:

\[-16t^2 + 45 = -3t + 32\]

Shuffle that around a bit (subtract \(-3t - 32\) from both sides) and you get:

\[-16t^2 + 3t + 13 = 0\]

There's your quadractic equation. If you stick this into the quadratic formula, you'll get \(t = 1\) or \(-13/16\). We can forget about negative time, so \(t = 1\) is the one that matters. That's the time at which the squirrel intercepts the acorn.

To convert that back into height, plug it into either of your original equations. (A good way to check your work is to put it into both and make sure they match.)

\[h = -3t + 32\]
\[= -3 + 32\]
\[= 29\]

They intercept at 29 feet, in one second.

---

**Systems of Equations and Inequalities**
The student applies mathematical processes to formulate systems of equations and inequalities, use a variety of methods to solve and analyze reasonableness of solutions. The student is expected to:

| 3(D) determine the reasonableness of solutions to systems of a linear equation and a quadratic equation in two variables; |

**Unpacking**
What does this standard mean that a student will know and be able to do?

Determine
When solving quadratic equations, 2 solutions are expected. However, both
solutions will not always make sense. The students need to understand ideas like negative time doesn’t exist, and negative distance is invalid. Students should have the opportunity to **determine** whether a solution is **reasonable**. This would be a great forum topic that would give the kids a chance to discuss together online.

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<td>What does this standard mean that a student will know and be able to do?</td>
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<td>3(E) formulate systems of at least two linear inequalities in two variables;</td>
<td>Formulate</td>
</tr>
<tr>
<td></td>
<td>1) <strong>Formulate</strong> a system of inequalities for the following application: Nola’s Nuts has 90 pounds of cashews and 120 pounds of peanuts available. These are to be mixed in 12 ounce packages as follows: Mix Yum: Contains 8 ounces of peanuts and 4 ounces of cashews Mix Tasty: Contains 6 ounces of peanuts and 6 ounces of cashews. *16 oz = 1 pound</td>
</tr>
<tr>
<td></td>
<td>2) <strong>Formulate</strong> a system of inequalities for the following application: Mike’s Toy Truck Company manufactures two models of toy trucks, standard model and deluxe model. Each standard model requires 2 hours for painting and 3 hours for detail work; each deluxe model requires 3 hours of painting and 4 hours of detail work. Two painters and three detail workers are employed by the company and each works 40 hours per week.</td>
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### Systems of Equations and Inequalities

**The student applies mathematical processes to formulate systems of equations and inequalities, use a variety of methods to solve and analyze reasonableness of solutions. The student is expected to:**

<table>
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<tr>
<th><strong>3(F)</strong> solve systems of two or more linear inequalities in two variables; and</th>
<th><strong>Solve</strong> The students will <strong>solve</strong> problems like the one above.</th>
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</table>

**Unpacking**

What does this standard mean that a student will know and be able to do?

### Systems of Equations and Inequalities

**The student applies mathematical processes to formulate systems of equations and inequalities, use a variety of methods to solve and analyze reasonableness of solutions. The student is expected to:**

| **3(G)** determine possible solutions in the solution set of systems of two or more linear inequalities in two variables. | **Determine** The students can **determine** whether a solution is part of the solution set in two ways:
1) graph the system of inequalities and **determine** if the solution lies within the shaded area, or
2) plug the values into the inequalities and **determine** if they create a true statement. If the statement is true, the values are part of the solution set, if the statement is not true, the values are not part of the solution set. |
|---|---|

**Unpacking**

What does this standard mean that a student will know and be able to do?

### Quadratic and Square Root Functions, Equations, and Inequalities.

**Unpacking**

What does this standard mean that a student
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<th>The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:</th>
<th>will know and be able to do?</th>
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<td>4(A) write the quadratic function given three specified points in the plane;</td>
<td>Write</td>
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<td>This is a good explanation of how to write the equation of a quadratic function given three specified points in the plane: <a href="http://mrpilarski.wordpress.com/category/algebra-2/equations-and-graphs/parabolas/finding-the-equation-given-three-points/">http://mrpilarski.wordpress.com/category/algebra-2/equations-and-graphs/parabolas/finding-the-equation-given-three-points/</a></td>
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**Quadratic and Square Root Functions, Equations, and Inequalities.**

The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

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<th>4(B) write the equation of a parabola using given attributes, including vertex, focus, directrix, axis of symmetry, and direction of opening;</th>
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<td>What does this standard mean that a student will know and be able to do?</td>
<td>Write</td>
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<td><strong>Example:</strong> If the focus of a parabola is (2, 5) and the directrix is ( y = 3 ), find the equation of the parabola. Let ((x_0, y_0)) be any point on the parabola. Find the distance between ((x_0, y_0)) and the focus. Then find the distance between ((x_0, y_0)) and directrix. Equate these two distance equations and the simplified equation in (x_0) and (y_0) is equation of the parabola. The distance between ((x_0, y_0)) and (2, 5) is (\sqrt{(x_0 - 2)^2 + (y_0 - 5)^2}). The distance between ((x_0, y_0)) and the...</td>
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directrix, \( y = 3 \) is 
\[ |y_0 - 3| \].
Equate the two distance expressions and square on both sides.
\[
\sqrt{(x_0 - 2)^2 + (y_0 - 5)^2} = |y_0 - 3|
\]
\[
(x_0 - 2)^2 + (y_0 - 5)^2 = (y_0 - 3)^2
\]
Simplify and bring all terms to one side:
\[
x_0^2 - 4x_0 - 4y_0 + 20 = 0
\]
Write the equation with \( y_0 \) on one side:
\[
y_0 = \frac{x_0^2}{4} - x_0 + 5
\]
This equation in \((x_0, y_0)\) is true for all other values on the parabola and hence we can rewrite with \((x, y)\).
So, the equation of the parabola with focus \((2, 5)\) and directrix is \( y = 3 \) is
\[
y = \frac{x^2}{4} - x + 5
\]

**Quadratic and Square Root Functions, Equations, and Inequalities.**
The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

4(C) determine the effect on the graph of \( f(x) = \sqrt{x} \) when \( f(x) \) is replaced by \( af(x) \), \( f(x) + d \), \( f(bx) \), and \( f(x - c) \) for specific positive and negative values of \( a \), \( b \), \( c \), and \( d \);

**Unpacking**
What does this standard mean that a student will know and be able to do?

**Example 1: Transforming a Square Root Function Vertically**
What are the graphs of \( y = \sqrt{x} - 2 \) and \( y = \sqrt{x} + 1 \)?
The graph of \( y = \sqrt{x} - 2 \) is the graph of \( y = \sqrt{x} \) shifted down 2 units.
The graph of \( y = \sqrt{x} + 1 \) is the graph of \( y = \sqrt{x} \) shifted up 1 unit.

Example 2: Transforming a Square Root Function Horizontally
What are the graphs of \( y = \sqrt{x} + 4 \) and \( y = \sqrt{x} - 1 \)?

The graph of \( y = \sqrt{x} + 4 \) is the graph of \( y = \sqrt{x} \) shifted left 4 units.

The graph of \( y = \sqrt{x} - 1 \) is the graph of \( y = \sqrt{x} \) shifted right 1 unit.

Example 3:
What is the graph of \( y = -\frac{1}{2} \sqrt{x - 3} + 1 \)?

Step 1: Choose several points from the parent function \( y = \sqrt{x} \).

Step 2: Multiply the y-coordinate by \(-\frac{1}{2}\). This shrinks the parent function by the factor of \( \frac{1}{2} \) and reflects the result in the x-axis.

Step 3: Translate the graph from step 2 to the right 3 units and up 1 unit.
# Quadratic and Square Root Functions, Equations, and Inequalities.

The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

### Unpacking

What does this standard mean that a student will know and be able to do?

### 4(D) transform a quadratic function 

\[ f(x) = ax^2 + bx + c \]  

\[ f(x) = a(x - h)^2 + k \]  

to the form

Write in vertex form.

\[ y = x^2 + 12x + 32 \]

\[ y - 32 = x^2 + 12x \]

\[ y - 32 + 36 = x^2 + 12x + 36 \] [Note: \( \left( \frac{12}{2} \right)^2 = 36 \)]

\[ y + 4 = x^2 + 12x + 36 \]

\[ y + 4 = (x + 6)(x + 6) \]

\[ y + 4 = (x + 6)^2 \]

\[ y = (x + 6)^2 - 4 \]
student is expected to:

<table>
<thead>
<tr>
<th>4(E) formulate quadratic and square root equations using technology given a table of data;</th>
<th>Formulate Students should be given several different tables of values (both quadratic and exponential tables) and asked to input the tables in their graphing calculator to view the different graphs. Example:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y_1$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>

Make sure that both STAT PLOTs are turned on; one for L1 and L2 and the other for L1 and L3.

Students should then recognize the difference in the graphs. From these basic graphs and tables students should be guided to formulate parent functions without a calculator. From there, change the x-value ($y_1$), which will in turn change the endpoint of both functions. Students can then use the calculator to formulate the equation of more complex tables.
**Quadratic and Square Root Functions, Equations, and Inequalities.**
The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

4(F) solve quadratic and square root equations;

**Unpacking**
What does this standard mean that a student will know and be able to do?

Solve

Students should be able to solve any quadratic equation by completing the square, factoring, or quadratic formula. Square root equations should be approached the same as any other equation, isolating the radical and then squaring both sides of the equation.

**Example 1 (Square Root):**
\[
\sqrt{x + 3} + 2 = 4 \\
\sqrt{x + 3} = 2 \\
x + 3 = 4 \\
x = 1
\]

**Example 2 (Quadratic):**
f(x) = 2x^2 − 9x − 5
0 = 2x^2 − 9x − 5
0 = (2x + 1)(x − 5)
x = −1/2 and 5

**Example 3:**
\[
\sqrt{x + 2} = x - 1 \\
x + 2 = (x - 1)^2 \\
x + 2 = x^2 - 2x + 1 \\
0 = x^2 - 3x - 1
\]
\[
x = \frac{3 \pm \sqrt{9-4(1)(-1)}}{2}
\]
\[
x = \frac{3 \pm \sqrt{13}}{2}
\]

Example 3 has both quadratic and square root properties and uses the quadratic formula to solve for \(x\).

**Quadratic and Square Root Functions, Equations, and Inequalities.**

The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

- 4(G) identify extraneous solutions of square root equations; and

**Unpacking**

What does this standard mean that a student will know and be able to do?

**Identify**

When solving for \(x\), the answers must be plugged back into the original equation to check for validity.

**Example:**

<table>
<thead>
<tr>
<th>Check (x = 3)</th>
<th>Check (x = 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{2x-5} + 4 = x)</td>
<td>(\sqrt{2x-5} + 4 = x)</td>
</tr>
<tr>
<td>(\sqrt{2(3)-5} + 4 = 3)</td>
<td>(\sqrt{2(7)-5} + 4 = 7)</td>
</tr>
<tr>
<td>(\sqrt{6-5} + 4 = 3)</td>
<td>(\sqrt{14-5} + 4 = 7)</td>
</tr>
<tr>
<td>(\sqrt{1} + 4 = 3)</td>
<td>(\sqrt{9} + 4 = 7)</td>
</tr>
<tr>
<td>(1 + 4 = 3)</td>
<td>(3 + 4 = 7)</td>
</tr>
<tr>
<td>(5 = 3 \times)</td>
<td>(? = 7 \checkmark)</td>
</tr>
</tbody>
</table>

**Quadratic and Square Root Functions, Equations, and Inequalities.**

**Unpacking**

What does this standard mean that a student
The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

<table>
<thead>
<tr>
<th>4(H)</th>
<th>solve quadratic inequalities.</th>
</tr>
</thead>
</table>

Solve

Quadratic inequalities should be treated and solved the same as a linear equation, as far as how to isolate the variable. Once the radical is isolated, square both sides of the inequality to isolate the variable.

Example:

\[ 3\sqrt{x} + 2 \geq 8 \]
\[ 3\sqrt{x} \geq 6 \]
\[ \sqrt{x} \geq 2 \]
\[ x \geq 4 \]

Students need to understand that the same rule applies to solving radical inequalities as linear inequalities—If you multiply or divide by a negative number when isolating the radical or the variable, the inequality symbol flips.

---

Exponential and Logarithmic Functions and Equation.
The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:

<table>
<thead>
<tr>
<th>5(A)</th>
<th>determine the effects on the key attributes on the graphs of ( f(x) = b^x ) and ( f(x) = \log_{b}x ) where ( b ) is 2, 10, and e when</th>
</tr>
</thead>
</table>

Unpacking

What does this standard mean that a student will know and be able to do?
<table>
<thead>
<tr>
<th>Exponential and Logarithmic Functions and Equation.</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:</td>
<td>What does this standard mean that a student will know and be able to do?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5(B) formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation;</th>
<th>Solve</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Exponential and Logarithmic Functions and Equation.</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>5(C) rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations;</th>
<th>Solve</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Exponential and Logarithmic Functions and Equation.</th>
<th>Unpacking</th>
</tr>
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<tbody>
<tr>
<td>The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:</td>
<td>What does this standard mean that a student will know and be able to do?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5(D) solve exponential equations of the form</th>
<th>Solve</th>
</tr>
</thead>
</table>
\[ y = ab^x \] where \( a \) is a nonzero real number and \( b \) is greater than zero and not equal to one and single logarithmic equations having real solutions;

### Exponential and Logarithmic Functions and Equation.
The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:

5(E) determine the reasonableness of a solution to a logarithmic equation.

### Unpacking
What does this standard mean that a student will know and be able to do?

Determine

Logarithmic equations cannot have a negative argument, therefore all solutions to logarithmic equations must be plugged back into the argument to assess the validity of the solution.

Example:
\[ \log_{10}(x - 5) = 1 \]
\[ 10 = x - 5 \]
\[ x = 15 \]

Substitute back in:
\[ \log_{10}(15 - 5) = 1 \]

Since the argument \((15 - 5)\) is a positive number, the solution to the logarithmic equation is reasonable.

### Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.
The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and
inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

6(A) analyze the effect on the graphs of \( f(x) = x^3 \) and \( f(x) = 3\sqrt{x} \) when \( f(x) \) is replaced by \( af(x) \), \( f(bx) \), \( f(x - c) \), and \( f(x) + d \) for specific positive and negative real values of \( a \), \( b \), \( c \), and \( d \);

Example 1: Vertical and Horizontal Transformations of Cubic functions

Use the graph of \( f(x) = x^3 \) to graph \( g(x) = (x - 2)^3 \) and \( h(x) = x^3 + 4 \).

The graph of \( g(x) = (x - 2)^3 \) is the graph of \( f(x) = x^3 \) shifted down 2 units.

The graph of \( h(x) = x^3 + 4 \) is the graph of \( f(x) = x^3 \) shifted right 4 units.

Example 2:

Step 1: Choose several points from the parent function \( f(x) = \sqrt[3]{x} \).

Step 2: Multiply the \( y \)-coordinate by \(-2\). This stretches the parent function by the factor of 2 and reflects the result in the \( x \)-axis.

Step 3: Translate the graph from step 2 to the left 5 units and down 2 units.
The graph of \( g(x) = -\sqrt[3]{x+5} - 2 \) is the graph of \( f(x) = \sqrt[3]{x} \) shifted down 2 units.

<table>
<thead>
<tr>
<th>Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.</th>
<th>Unpacking</th>
<th>6(B) solve cube root equations that have real roots;</th>
</tr>
</thead>
</table>
| The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to: | What does this standard mean that a student will know and be able to do? | 2 \cdot \sqrt[3]{3x} + 3 = x \\
2 \cdot \sqrt[3]{3x} = x - 3 \\
8(3x) = (x - 3)^3 \\
24x = x^3 - 9x^2 + 27x - 27 \\
x^3 - 9x^2 + 3x - 27 = 0 \\
x^2(x - 9) + 3(x - 9) = 0 \\
(x^2 + 3)(x - 9) = 0 \\
x = 9 is the solution (x^2 + 3) does not give a root because the square root of a negative number is not a real number. |
Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.
The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

6(C) analyze the effect on the graphs of $f(x) = |x|$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x-c)$, and $f(x) + d$ for specific positive and negative real values of $a$, $b$, $c$, and $d$;

Unpacking
What does this standard mean that a student will know and be able to do?

Analyze
Example 1: Vertical and Horizontal Transformations of Absolute Value functions
Use the graph of $f(x) = |x|$ to graph $g(x) = |x + 3|$ and $h(x) = |x| + 2$.

The graph of $h(x) = |x| + 2$ is the graph of $f(x) = |x|$ shifted up 2 units.

The graph of $g(x) = |x + 3|$ is the graph of $f(x) = |x|$ shifted 3 units to the left.

Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.
The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

Unpacking
What does this standard mean that a student will know and be able to do?
value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

- **6(D)** formulate absolute value linear equations;

### Example:
The solutions of \(|x| = 3\) are the two points that are 3 units from zero. The solution is a disjunctions: \(x = 3\) or \(x = -3\).

---

**Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.**
The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

- **6(E)** solve absolute value linear equations;

### Unpacking
What does this standard mean that a student will know and be able to do?

### Solving an Absolute Value Equation
Solve and graph: \(|2x - 1| = 5\)

1. \(|2x - 1| = 5\)
2. \(2x - 1 = 5\) or \(2x - 1 = -5\)
3. \(2x = 6\) or \(2x = -4\)
4. \(x = 3\) or \(x = -2\)

Add 1 to each side of both equations. Divide each side of both equations by 2.

Check:
- \(|2(3) - 1| \geq 5\) or \(|2(-2) - 1| \geq 5\)
- \(|6 - 1| \geq 5\) or \(|-4 - 1| \geq 5\)
- \(|5| = 5\) or \(|-5| = 5\)
Solving a Multi-Step Absolute Value Equation

Solve and graph:
3 \( |x + 2| - 1 = 8 \)

\[
\begin{align*}
3|x + 2| - 1 &= 8 \\
3|x + 2| &= 9 \\
|x + 2| &= 3
\end{align*}
\]

Add 1 to each side.

\[
\begin{align*}
x + 2 &= 3 \\
x &= 1 \\
\text{or} \\
x + 2 &= -3 \\
x &= -5
\end{align*}
\]

Divide each side by 3.

Rewrite as two equations.

Subtract 2 from each side of both equations.

Check: 3 \((1) + 2\) = 1 \(\not\leq\) 8
3 \((-5) + 2\) = -1 \(\not\leq\) 8
8 = 8 \(\checkmark\)
8 = 8 \(\checkmark\)

Extraneous Solutions
What is the solution \( |3x + 2| = 4x + 5 \)

Check for extraneous solutions.

\[
\begin{align*}
|3x + 2| &= 4x + 5 \\
3x + 2 &= 4x + 5 \\
-x &= 3 \\
x &= -3
\end{align*}
\]

or

\[
\begin{align*}
3x + 2 &= -4x - 5 \\
7x &= -7 \\
x &= -1
\end{align*}
\]

Rewrite as two equations.

Solve each equation.

Check:
3 \((-3) + 2\) \(\not\leq\) 4 \((-3) + 5\)
3 \((-1) + 2\) \(\leq\) 4 \((-1) + 5\)
9 + 2 \(\neq\) 12 + 5
\(-3 + 2\) \(\leq\) -4 + 5
\[-7\] \(\neq\) -7
\[-1\] \(\leq\) 1

Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.
The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

6(F) solve absolute value linear inequalities;

Unpacking
What does this standard mean that a student will know and be able to do?

Solving Absolute Value Inequality \( |x| < a \)

\[
x > -a \text{ and } x < a \text{ or } -a < x < a
\]
Example: The solutions of $|x| < 3$ are the points that are less than 3 units from zero. The solution is the conjunction: $-3 < x < 3$.

Solving the Absolute Value Inequality ($|x| < a$)

Solve and Graph:

$|2x - 1| < 5$

$-5 < 2x - 1 < 5$  $2x - 1$ is between $-5$ and $5$.

$-4 < 2x < 6$  Add $1$ to each part.

$-2 < x < 3$  Divide each part by $2$.

Solving Absolute Value Inequality $|x| > a$

$x < -a$ OR $x > a$

Example: The solutions of $|x| > 3$ are the points that are more than 3 units from zero. The solution is the disjunction: $x < -3$ OR $x > 3$.

Solving the Absolute Value Inequality ($|x| > a$)

Solve and graph:

$|2x + 4| > 6$

$2x + 4 \leq -6$  OR  $2x + 4 \geq 6$  Rewrite as a compound inequality.

$2x \leq -10$  OR  $2x \geq 2$  Subtract $4$ from each side of both inequalities.

$x \leq -5$  OR  $x \geq 1$  Divide each side of both inequalities by $2$.
<table>
<thead>
<tr>
<th><strong>Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.</strong></th>
<th><strong>Unpacking</strong></th>
<th><strong>6(G) analyze the effect on the graphs of f(x) = 1/x when f(x) is replaced by af(x), f(bx), f(x-c), and f(x) + d for specific positive and negative real values of a, b, c, and d:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:</td>
<td>What does this standard mean that a student will know and be able to do?</td>
<td>Example: As a result of the transformation of the graph of $y = x^3$ into the graph of $y - 4 = (x - 3)^3$, point (3, 27) becomes point (6, $y$). The value of $y$ is __________. Solution: 31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y - 4 = (6 - 3)^3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y - 4 = 27$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$y = 31$</td>
</tr>
<tr>
<td></td>
<td>or</td>
<td>$y = (x - 3)^3 + 4$ is $y = x^3$ with a horizontal shift of 3 units right and a vertical shift of 4 units up. Therefore, 27 becomes 31.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.</strong></th>
<th><strong>Unpacking</strong></th>
<th><strong>6(H) formulate rational equations that model real-world situations:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:</td>
<td>What does this standard mean that a student will know and be able to do?</td>
<td>A kayaker spends an afternoon paddling on a river. She travels 3 miles upstream and 3 miles downstream in a total of 4 hours. In still water, the kayaker can travel 2 miles per hour.</td>
</tr>
</tbody>
</table>
Based on this information, what is the average speed of the river’s current? Is your answer reasonable?

The answer will be the average speed of the current. Let \( c \) represent the speed of the current. When the kayaker is going upstream, her speed is equal to still water minus \( c \). When the kayaker is going downstream, her speed is equal to still water plus \( c \).

<table>
<thead>
<tr>
<th>Distance (mi)</th>
<th>Average Speed (mi/h)</th>
<th>Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up</td>
<td>3</td>
<td>( \frac{3}{2 - c} )</td>
</tr>
<tr>
<td>Down</td>
<td>3</td>
<td>( \frac{3}{2 + c} )</td>
</tr>
</tbody>
</table>

Solve

\[
\begin{align*}
4(2 - c)(2 + c) &= \frac{3}{2 - c} \quad \text{(The LCD is (2 - c)(2 + c))} \\
4(2 - c)(2 + c) &= 3(2 + c) + 3(2 - c) \\
16 - 4c^2 - 6c + 6 - 3c &= 0 \\
16 - 4c^2 &= -12 \\
-4c^2 &= -24 \\
c &= \pm 3
\end{align*}
\]

The speed of the current cannot be negative. Therefore the average speed of the current is 1 mi/h.

---

Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.

The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

Unpacking

What does this standard mean that a student will know and be able to do?

6(I) solve rational equations that have real solutions;

What are the solutions of the rational equation?
Extraneous Solutions

Solve: \( \frac{x}{x-3} + \frac{x}{x+3} = \frac{2}{(x + 3)(x-3)} \)

\[ \begin{align*}
(x-3)(x+3) & \left( \frac{x}{x-3} + \frac{x}{x+3} \right) = 2 \\
(x-3)(x+3) \frac{x}{x-3} + (x-3)(x+3) \frac{x}{x+3} & = 2 \\
(x-3)(x+3) \frac{x}{x-3} + (x-3)(x+3) \frac{x}{x+3} & = 2 (x-3)(x+3) \\
(x-3)(x+3)x + (x-3)(x+3)x & = 2(x-3)(x+3) \\
x^2 + 3x + x^2 - 3x & = 2 \\
x^2 & = 2 \\
x & = \pm \sqrt{2}
\end{align*} \]

The solutions are \( x = 1 \) and \( x = -1 \).

Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.

The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

6(J) determine the reasonableness of a solution to a rational equation;

Unpacking

What does this standard mean that a student will know and be able to do?

A kayaker spends an afternoon paddling on a river. She travels 3 miles upstream and 3 miles downstream in a total of 4 hours. In still water, the kayaker can travel 2 miles per hour.
Based on this information, what is the average speed of the river’s current? Is your answer reasonable? (See plan and solution in the example for standard 6(H))

**Reasonableness of the Solution:**
The speed of the current cannot be negative. Therefore the average speed of the current is 1 mi/h.

---

**Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.**
The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

**Unpacking**
What does this standard mean that a student will know and be able to do?

**6(K) determine the asymptotic restrictions on the domain of a rational function and represent domain and range using interval notation, inequalities, and set notation; and**

**Determine**

1. Given \( f(x) = \frac{2}{x-3} \) what are all possible values for the domain \((x)\)?

Answer: If you plug 3 in for the x-value, the denominator will become zero, which causes the function to be undefined. So, our domain is everything except 3, written as \((\infty, 3) \cup (3, \infty)\) OR you could write it as \(x \neq 3\).

\(x=3\) would also be considered an asymptotic restriction.
**Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities.**

The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

<table>
<thead>
<tr>
<th>6(L) formulate and solve equations involving inverse variation.</th>
<th>Unpacking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Formulate</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Example 1:</strong></td>
<td></td>
</tr>
<tr>
<td>Equation: ( y = \frac{60}{x} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( x )</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td><strong>Describe the function</strong> ( y = \frac{60}{x} ).</td>
<td></td>
</tr>
<tr>
<td><strong>Answer:</strong></td>
<td></td>
</tr>
<tr>
<td>( y ) is inversely proportional to ( x ).</td>
<td></td>
</tr>
<tr>
<td>Doubling ( x ) causes ( y ) to halve. The product of ( x ) and ( y ) is always 60.</td>
<td></td>
</tr>
<tr>
<td><strong>Example 2:</strong></td>
<td></td>
</tr>
<tr>
<td>Part A: In a formula, ( Z ) varies inversely as ( p ). If ( Z ) is 200 when ( p = 4 ), find ( Z ) when ( p = 10 ). Set up the formula.</td>
<td></td>
</tr>
</tbody>
</table>
Part B: Find the missing constant, k, by using the first set of data given.

\[
200 = \frac{k}{4}
\]

Answer: \( k = 800 \)

<table>
<thead>
<tr>
<th>Number and Algebraic Methods</th>
<th>Unpacking</th>
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<tbody>
<tr>
<td>The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:</td>
<td>What does this standard mean that a student will know and be able to do?</td>
</tr>
<tr>
<td>7(A) add, subtract, and multiply complex numbers;</td>
<td>Add, Subtract, Multiply</td>
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</table>

**Example 1: Addition**

\[(5 + 2i) + (7 + 12i)\]
\[(5 + 7) + (2i + 12i)\]
\[12 + 14i\]

**Example 2: Subtract**

\[(8 + 6i) - (5 + 2i)\]
\[(8 + 6i) + (-5 - 2i)\]
\[(8 + -5) + (6i - 2i)\]
\[3 + 4i\]

**Example 3: Multiply**

**A** \((3i)(-5 + 2i)\)
\[-15i + 6i^2\]
\[-15i + 6i(-1)\]
\[-6 - 15i\]

**B** \((4 + 3i)(-1 - 2i)\)
\[-4 - 8i - 3i - 6i^2\]
\[-4 - 8i - 3i - 6(-1)\]
\[2 - 11i\]

**C** \((-6 + i)(-6 - i)\)
\[36 + 6i - 6i - i^2\]
\[36 + 6i - 6i - (-1)\]
\[37\]
**Number and Algebraic Methods**  
The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:

<table>
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<tr>
<th>Unpacking</th>
<th>7(B) add, subtract, and multiply polynomials;</th>
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<td>What does this standard mean that a student will know and be able to do?</td>
<td>Add, Subtract, Multiply</td>
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</table>

**Example 1: Addition**

\[(2x + 5y) + (3x - 2y)\]
\[= 2x + 5y + 3x - 2y\]
\[= 2x + 3x + 5y - 2y\]
\[= 5x + 3y\]

**Example 2: Subtraction**

\[x^3 + 3x^2 + 5x - 4 - (3x^3 - 8x^2 - 5x + 6)\]
\[= x^3 + 3x^2 + 5x - 4 - 3x^3 + 8x^2 + 5x - 6\]
\[=- 2x^3 + 11x^2 + 10x - 10\]

**Example 3: Multiplication**

\[(x - 2)(-x^2 + 3x + 1)\]
\[= x(-x^2) + x(3x) + x(1) - 2(-x^2) - 2(3x) - 2(1)\]
\[=-x^3 + 3x^2 + x + 2x^2 - 6x - 2\]
\[=-x^3 + 5x^2 - 5x - 2\]
7(C) determine the quotient of a polynomial of degree three and of degree four when divided by a polynomial of degree one and of degree two;

Number and Algebraic Methods
The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:

7(D) determine the linear factors of a polynomial function of degree three and of degree four using algebraic methods;

Number and Algebraic Methods
The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:

7(E) determine linear and quadratic factors of a polynomial expression of degree three and of degree four, including factoring the sum and difference of two cubes and factoring by grouping;
Number and Algebraic Methods
The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:

7(F) determine the sum, difference, product, and quotient of rational expressions with integral exponents of degree one and of degree two;

Unpacking
What does this standard mean that a student will know and be able to do?

**Sum**

\[
\frac{2}{x} + \frac{3}{x^2} + \frac{1}{2x} = \frac{2x}{2x^2} + \frac{3}{2x^2} + \frac{1}{2x}
\]

multiply to get common denominator

\[
= \frac{4x}{2x^2} + \frac{6}{2x^2} + \frac{1x}{2x^2}
\]

add numerators

\[
= \frac{4x + 6 + 1x}{2x^2}
\]

\[
= \frac{5x + 6}{2x^2}
\]

**Product**

What is the product \(\frac{x^2 + x - 6}{x - 5} \cdot \frac{x^2 - 25}{x^3 + 4x + 3}\) in simplest form? State any restrictions on the variable.

\[
\frac{x^2 + x - 6}{x - 5} \cdot \frac{x^2 - 25}{x^3 + 4x + 3}
\]

Factor all polynomials.

\[
= \frac{(x + 3)(x - 2)}{x - 5} \cdot \frac{(x + 5)(x - 5)}{(x + 3)(x + 1)}
\]

\[
= \frac{(x - 2)(x + 5)}{x + 1}
\]

Divide out common factors.

Simplify.

The product is \(\frac{x^2 - 20}{x + 1}\) for \(x \neq -3, x \neq 5,\) and \(x \neq -1\). The restrictions \(x \neq -3\) and \(x \neq 5\) are evident from the simplified form, but are needed to prevent the denominators in the original product from being zero.

**Quotient**

Write

Think

To divide, you multiply by the reciprocal.

\[
\frac{2 - x}{x^2 + 2x + 1} \div \frac{x^2 + 3x - 10}{x - 1}
\]

\[
= \frac{2 - x}{x^2 + 2x + 1} \cdot \frac{x - 1}{x^2 + 3x - 10}
\]

\[
= \frac{2 - x}{(x + 1)(x + 1)} \cdot \frac{(x + 1)(x - 1)}{(x + 5)(x - 2)}
\]

\[
= \frac{2 - x}{(x + 1)(x + 5)}
\]

The expressions may have common factors. So, factor the numerators and denominators.

Factor \(-1\) from \((2 - x)\) to get a second \((x - 2)\).

Divide out common factors.

Rewrite the remaining factors.

Identify the restrictions from the denominator of the simplified expression and from any other denominator used.

\(x = -1, x \neq -5, x \neq 1,\) and \(x \neq 2\)
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<td><strong>7(G)</strong> rewrite radical expressions that contain variables to equivalent forms;</td>
<td>Example:</td>
</tr>
<tr>
<td>$\sqrt[4]{b^{33}} = \sqrt[4]{b^{20}b^3} = \sqrt[4]{b^{20}} \sqrt[4]{b^3} = b^5 \sqrt[4]{b^3}$</td>
<td></td>
</tr>
</tbody>
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<td><strong>7(H)</strong> solve equations involving rational exponents; and</td>
<td><strong>Example 1</strong></td>
</tr>
<tr>
<td>$(x - 3)^{\frac{1}{2}} = 3$</td>
<td>$\left[(x - 3)^{\frac{1}{2}}\right]^2 = 3^2$</td>
</tr>
<tr>
<td>$\quad (x - 3) = 9$</td>
<td>$\quad x - 3 = 9$</td>
</tr>
<tr>
<td>$\quad \frac{x}{3} = 3$</td>
<td>$\quad \frac{x}{3} = 3$</td>
</tr>
<tr>
<td>$\quad x - 12$</td>
<td></td>
</tr>
<tr>
<td>$(4x + 1)^{\frac{1}{3}} = 3$</td>
<td>$\left[(4x + 1)^{\frac{1}{3}}\right]^3 = 3^3$</td>
</tr>
<tr>
<td>$\quad (4x + 1) = 27$</td>
<td>$\quad 4x + 1 = 27$</td>
</tr>
<tr>
<td>$\quad 4x + 1 = 27$</td>
<td>$\quad 4x + 1 = 27$</td>
</tr>
<tr>
<td>$\quad 4x = 26$</td>
<td>$\quad 4x = 26$</td>
</tr>
<tr>
<td>$\quad x = 6.5$</td>
<td>$\quad x = 6.5$</td>
</tr>
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</table>
**Number and Algebraic Methods**  
The student applies mathematical processes to simplify and perform operations on expressions and to solve equations. The student is expected to:

| 7(I)  | write the domain and range of a function in interval notation, inequalities, and set notation. |

**Unpacking**  
What does this standard mean that a student will know and be able to do?

Students should know:

**Set Notation:**
- \( \{x \mid x < -2\} \) reads “the set of all \( x \) such that \( x \) is less than negative 2”.

**Interval Notation:**
- **Brackets** indicate that first or last number is in the set:
  - Example: \((-\infty, -2]\) means the same thing as \( x \leq -2 \)
  - Infinity (positive or negative) never gets a bracket
  - Always write the smaller number, bigger number

- **Parenthesis** indicate that first or last number is **not** in the set:
  - Example: \((-\infty, -2)\) means the same thing as \( x < -2 \)
  - Neither the negative infinity or the negative 2 are included in the interval
  - Always write the smaller number, bigger number

**Example:**
Write the domain and range in interval notation, inequalities and set notation.

![Graph showing domain and range with corresponding notations](image)

**Data**  
The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions. The student is expected to:

|  |  |

**Unpacking**  
What does this standard mean that a student will know and be able to do?
8 (A) analyze data to select the appropriate model from among linear, quadratic, and exponential models;

**Example:** Based on each table, identify the shape of the graph.

Data
The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions. The student is expected to:

**Unpacking**
What does this standard mean that a student will know and be able to do?
The student should know the following models:

**Linear Regression**
Quadratic Regression

\[ y = -0.91x^2 + 5.47x - 4.31 \]

Exponential Regression

\[ y = (1.01)^{0.99x} \]
8(B) use regression methods available through technology to write a linear function, a quadratic function, and an exponential function from a given set of data; and

Example:

The following table shows the number of students enrolled in public elementary schools in the US (Source: US Census Bureau). Make a scatterplot with the number of students as the dependent variable, and the number of years since 1990 as the independent variable. Find which curve fits this data the best and predict the school enrollment in the year 2007.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of students (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>26.6</td>
</tr>
<tr>
<td>1991</td>
<td>26.6</td>
</tr>
<tr>
<td>1992</td>
<td>27.1</td>
</tr>
<tr>
<td>1993</td>
<td>27.7</td>
</tr>
<tr>
<td>1994</td>
<td>28.1</td>
</tr>
<tr>
<td>1995</td>
<td>28.4</td>
</tr>
<tr>
<td>1996</td>
<td>28.1</td>
</tr>
<tr>
<td>1997</td>
<td>29.1</td>
</tr>
<tr>
<td>1998</td>
<td>29.3</td>
</tr>
<tr>
<td>2003</td>
<td>32.5</td>
</tr>
</tbody>
</table>

Solution

We need to perform linear, quadratic and exponential regression on this data set to see which function represents the values in the table the best.

**Step 1: Input the data.**

Input the values of \(x\) in the first column \(L_1\) and the values of \(y\) in the second column \(L_2\).

**Step 2: Draw the scatterplot.**

Set the window size: \(0 \leq x \leq 10\) and \(20 \leq y \leq 40\).

Here is the scatterplot:

![Scatterplot Image]

**Linear Regression**

The function of the line of best fit is \(y = 0.51x + 26.1\). Here is the graph of the function on the scatterplot:

![Linear Regression Graph Image]
From the graphs, it looks like the quadratic functions is the best fit for the data set.

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</tr>
<tr>
<td>8(C) predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.</td>
<td>Retail prescription drug sales in the United States increased from 1995 to 2000 as shown in the table.</td>
</tr>
</tbody>
</table>
Model the data using a quadratic function and use the model to predict the sales for 1996 and 2001.

Based on retail sales from 1995 to 2000, retail prescription drugs may be modeled by

\[ S(t) = 1.411t^2 + 7.411t + 68.55 \] billion dollars

where \( t \) is the number of years since 1995.

In 1996 \( t = 1 \) and in 2001 \( t = 6 \).

\[ S(1) = 1.411(1)^2 + 7.411(1) + 68.55 = 77.402 \]
\[ S(6) = 1.411(6)^2 + 7.411(6) + 68.55 = 163.992 \]

In 1996 drug sales were 77.402 billion and in 2001 drug sales were 163.992 billion.