7th Grade Texas Mathematics: Unpacked Content

What is the purpose of this document?
To increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand and be able to do. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the new math standards.

What is in the document?
Descriptions of what each standard means a student will know, understand, and be able to do. The “unpacking” of the standards done in this document is an effort to answer a simple question “What does this standard mean that a student must know and be able to do?” and to ensure the description is helpful, specific and comprehensive for educators.
At A Glance:

New to 7th Grade:

- Visual representation to describe relationships between sets of rational numbers
- Solve problems using addition, subtraction, multiplication and division of rational numbers (8th grade)
- multiple representations of constant rates (8th grade)
- determine the constant of proportionality (Algebra I)
- solve problems with percent increase and decrease and financial literacy problems (8th grade)
- convert between measurement systems including the use of proportions and unit rates (8th grade)
- describe π as the ratio of the circumference of a circle to its diameter and determine area for circumference and area of a circle (6th grade)
- simulations for simple and compound events (8th grade)
- probability of simple event and its complement (6th grade)
- make inferences from sampling (8th grade)
- solve problems using data in bar graphs, dot plots and circle graphs (6th)
- solve problems using qualitative and quantitative predictions
- linear relationships using verbal descriptions, tables, graphs, and equations: y=mx+b (8th grade)
- volume comparisons of prisms and pyramids (8th grade)
- lateral & total surface area of prisms and pyramids (8th grade)
- model, write, solve, graph inequalities (Algebra I)
- write and solve equations using geometry concepts, including the sum of angles in a triangle
- compare and use data in random samples from populations
- analyze shapes, center, and spread of comparative dot plots and box plots
- Financial Literacy: income tax; personal budget; financial assets and liabilities; budgets; net worth

Moved from 7th Grade:

- convert between frac/dec/percents
- squares & square roots
- model multiplying and dividing fractions & decimals
- compare, order, model, solve integers
- order of operations & exponents
- arithmetic sequences
- classify angles pairs, triangles, quadrilaterals, 3-D figures
- coordinate plane
- graph reflections and translations
- sketch 3-D figures with different views
- describe and justify data using mean, median, mode and range

**Instructional Implications for 2013-14:**

- Sometime between 2013/14 6th gr current TEKS and 2014/15 7th gr new TEKS, students will need to have instruction on multiplying and dividing fractions and decimals and all operations with integers
- Including inequalities as well as equations
- All probability instruction happens at 7th grade
- Measurements of circular 2-D figures are introduced in 7th grade (no longer 6th)

**Professional Learning Implications for 2013-14:**

- Learning the new verbage and structure of TEKS (ex. Measurement strand isn't formulas anymore, its data...)
- PD and resources regarding Personal Financial Literacy
- PD related to the Data and Measurement strand, related to the new terminology and approach to data analysis
- Teachers will need to identify the gaps that will need to be addressed in the 2013-14 school year.
- Embed the process standards into instruction and application
- Identify academic vocabulary
- Initial learning of the teachers' grade level TEKS (teachers unpacking the TEKS at their grade level)
- Vertical study of the strands to know how the TEKS align and progress from 6th through 8th grade
Grade 7 Primary Focal Areas:

The Primary Focal Areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction.

The primary focal areas in Grade 7 are number and operations; proportionality; expressions, equations, and relationships; and measurement and data. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use concepts of proportionality to explore, develop, and communicate mathematical relationships, including number, geometry and measurement, and statistics and probability. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other. Students connect verbal, numeric, graphic, and symbolic representations of relationships, including equations and inequalities. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, and reasoning to draw conclusions, evaluate arguments, and make recommendations. While the use of all types of technology is important, the emphasis on algebra readiness skills necessitates the implementation of graphing technology.

Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;
(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
(E) create and use representations to organize, record, and communicate mathematical ideas;
(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
### Number and Operations: TEKS: 7.2

#### 7.2(A) Extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of rational numbers.

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<table>
<thead>
<tr>
<th>The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>In 6th grade, students “classify” sets of numbers (whole, integers, and rational numbers). In 7th grade, the verb changes to “describe relationships” between sets of rational numbers. Students need to have experiences using visual representations (such as a Venn diagrams) to classify number systems.</td>
</tr>
</tbody>
</table>

#### Example 1:
How are integers and whole numbers alike? different?

#### Example 2:
Name all the sets to which each number belongs.

\[
0, \quad \frac{16}{3}, \quad \frac{-8}{4}, \quad 8, \quad 61, \quad -\sqrt{25}, \quad \sqrt{49}, \quad 5.3
\]

#### Misconceptions:
1. Knowing only a limited number of models for interpreting fractions; for example, not recognizing fractions as locations on a number line or as division calculations.
2. Understanding the symbol “-” as subtraction and as a negative.
Number and Operations:
TEKS: 7.3

<table>
<thead>
<tr>
<th>TEKS: 7.3</th>
<th>The student applies mathematical process standards to add, subtract, multiply, and divide while solving problems and justifying solutions. The student is expected to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3(A) Add, subtract, multiply, and divide rational numbers fluently</td>
<td>Extend the fluent computation with positive rational numbers from 6th grade to 7th grade by now including all rational numbers (includes negative). Apply rational number operations in problem situations. Students will need to have a solid understanding of integer rules to be able to apply to rational numbers. While students need to be fluent in operations with both positive and negative rational numbers, the application problems will may tend to focus on positive numbers for meaningful examples, such as change in temperature, stock market loss and gains.</td>
</tr>
<tr>
<td>7.3(B) Apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers.</td>
<td>Example 1: A recipe that makes 1.5 dozen cookies requires 0.75 cups of sugar. Shanika wants to make 6 dozen cookies. How many cups of sugar will she need?</td>
</tr>
<tr>
<td></td>
<td>Example 2: Ocean water freezes at about −2 ½ degrees C. Fresh water freezes at 0 degrees C. Antifreeze, a liquid used to cool most car engines, freezes at −64 degrees C. Imagine that the temperature is exactly at the freezing point for ocean water. How many degrees must the temperature drop for the antifreeze to turn to ice?</td>
</tr>
<tr>
<td></td>
<td>Misconceptions: 1. Students may believe that all rational numbers are positive. 2. Students have trouble using models to illustrate operations or to connect operations on objects to algorithms.</td>
</tr>
</tbody>
</table>

Proportionality:
TEKS: 7.4

<table>
<thead>
<tr>
<th>TEKS: 7.4</th>
<th>The student applies mathematical process standards to represent and solve problems involving proportional relationships.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.4(A) Represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including ( d = rt )</td>
<td>Students’ understanding of the multiplicative reasoning used with proportions continues from 6th grade. Students will need to learn the vocabulary related to these TEKS: direct variation, constant of proportionality ( (k) ), etc. Students understand that the constant of proportionality is the unit rate. Students identify this amount from pictorial, tabular, verbal, numeric, graphical, and algebraic expressions. Students understand that the equation ( y = kx ) is read “( y ) varies directly as ( x ).” Students understand that ( d = rt ) is a proportional relationship and a specific example of ( y = kx )</td>
</tr>
<tr>
<td>7.4(B) Calculate unit rates from rates in</td>
<td></td>
</tr>
</tbody>
</table>

7.4(C) Determine the constant of proportionality ($k = y/x$) within mathematical and real-world problems

**Example 1:**
The graph below represents the price of the bananas at one store. What is the constant of proportionality?

![Graph of Cost of Bananas](image)

From the graph, it can be determined that 4 pounds of bananas is $1; therefore, 1 pound of bananas is $0.25, which is the constant of proportionality for the graph.

Students need to know that any point on the graph will give them the constant of proportionality. $K = y/x$ denotes that $K = 0.25/1$ or $1.00/4$ or $1.50/6$

All will give 0.25 as the constant of proportionality; $y = 0.25x$

**Example 2:**
The price of bananas at another store can be determined by the equation: $P = 0.35n$, where $P$ is the price and $n$ is the number of pounds of bananas. What is the constant of proportionality (unit rate)?

Students need to be able to express this from words and equations. Constant of proportionality is 0.35 given the equation.

**Example 3:**
The table below gives the price for different numbers of books. Do the numbers in the table represent a proportional relationship?
Solution:
Students can examine the numbers to determine that the prices is the number of books multiplies by 3, except for 7 books. The row with seven books for $18 is not proportional to the other amounts in the tables; therefore, the table does NOT represent a proportional relationship.

If students had graphed the data in the table, they would see that it does not form a straight line, indicating that it is not proportional. Would they have seen this graphing only the first 3 sets of coordinates? No – they needed to graph (7,18)
There is no constant of proportionality since it is not proportional.

These TEKS are a pre-requisite for Algebra I and lead students to solving direct variation equations.
Misconceptions:
1. Sometimes students struggle with the conversion of words to symbols.

7.4(D) Solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease, and financial literacy problems

Students apply their previous knowledge of ratio, rate, and percent (single-step) problems from 6th grade to multi-step problems and financial literacy problems in 7th grade. Students need to know multiple strategies for solving proportion problems, such as cross products, equivalent ratios, unit rate, ratio tables, etc. Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Students use models to identify the parts of the problem and how the values are related. For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value. Students understand the use of proportional relationships is also extended to solve percent problems involving sales tax, markups and markdowns, simple interest (I = prt, where I = interest, p = principal, r = rate, and t = time (in years)), gratuities and commissions, fees, percent increase and decrease, and percent error.

Multistep Ratio/Rate::
- applying two unit rates within the same question (ie. mpg, mph)

Multistep Percent::
- finding the final cost after tax, tip, and/or discount
- percent of change (increase and decrease)
- discount off of a discounted price
- simple interest
- find tax rate when given the final cost and cost of an item before tax
- commission problems—finding commission, amount needed to sell to make a certain amount

Example 1:

A generator can run for 11 hours on 3 gallons of gasoline. If the gasoline costs $3 per gallon, which is closest to the cost per hour to run the generator?

Example 2:

A 24,000-gallon pool is being filled at a rate of 40 gallons per minute. At this rate, how many minutes will it take to fill this pool ¾ full?

Example 3:

Jamal needs to purchase a countertop for his kitchen. Jamal measured the countertop as 5 ft. The actual measurement is 4.5 ft. What is Jamal's percent error?

Solution:

\[
\% \text{ error} = \left| \frac{5 \text{ ft} - 4.5 \text{ ft}}{4.5} \right| \times 100
\]

\[
\% \text{ error} = \frac{0.5 \text{ ft}}{4.5} \times 100
\]

Example 4:

Games Unlimited buys video games for $10. The store increases their purchase price by 300%. What is the sales price of the video game?

Using proportional reasoning, if $10 is 100% then what amount would be 300%? Since 300% is 3 times 100%, $30 would be $10 times 3. Thirty dollars represents the amount of increase from $10 so the new price of the video game would be $40.
Example 5:
Gas prices are projected to increase by 124% by April 2015. A gallon of gas currently costs $3.80. What is the projected cost of a gallon of gas for April 2015?

Possible response: “The original cost of a gallon of gas is $3.80. An increase of 100% means that the cost will double. Another 24% will need to be added to figure out the final projected cost of a gallon of gas. Since 25% of $3.80 is about $0.95, the projected cost of a gallon of gas should be around $8.15.”

\[ $3.80 + 3.80 + (0.24 \times 3.80) = 2.24 \times 3.80 = 8.15 \]

Example 6:
A sweater is marked down 33% off the original price. The original price was $37.50. What is the sale price of the sweater before sales tax?

The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 \times \text{Original Price}.

Example 7:
A shirt is on sale for 40% off. The sale price is $12. What was the original price? What was the amount of the discount?
Solution:

<table>
<thead>
<tr>
<th>Discount</th>
<th>Sale Price - $12</th>
</tr>
</thead>
<tbody>
<tr>
<td>40% of original</td>
<td>60% of original price</td>
</tr>
<tr>
<td>Original Price (p)</td>
<td></td>
</tr>
</tbody>
</table>

The sale price is 60% of the original price. This reasoning can be expressed as \(12 = 0.60p\). Dividing both sides by 0.60 gives an original price of \$20\.

Example 8:

At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs by giving all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify the solution.

The sales team members need to sell the 48 and an additional 30% of 48. 14.4 is exactly 30% so the team would need to sell 15 more TVs than in April or 63 total (48 + 15)

Example 9:

A salesperson set a goal to earn \$2,000 in May. He receives a base salary of \$500 per month as well as a 10% commission for all sales in that month. How much merchandise will he have to sell to meet his goal?

\[ \text{Commission} = 0.10 \times \text{Sales} \]

\[ \text{Commission} \times 100\% = 10\% \times \text{Sales} \]

\[ 10\% \times \text{Sales} = \$1,500 \]

\[ \text{Sales} = \frac{\$1,500}{10\%} = \frac{\$1,500}{0.10} = \$15,000 \]

Example 10:

After eating at a restaurant, Mr. Jackson's bill before tax is \$52.50 The sales tax rate is 8%. Mr. Jackson decides to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip Mr. Jackson leaves for the waiter? How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill.
The amount paid = 0.20 x $52.50 + 0.08 x $52.50 = 0.28 x $52.50 or $14.70 for the tip and tax. The total bill would be $67.20.

Example 11:

Stephanie paid $9.18 for a pair of earrings. This amount includes a tax of 8%. What was the cost of the item before tax?

One possible solution path follows:

$9.18 represents 100% of the cost of the earrings + 8% of the cost of the earrings. This representation can be expressed as 1.08c = 9.18, where c represents the cost of the earrings. Solving for c gives $8.50 for the cost of the earrings.

Several problem situations have been represented with this standard; however, every possible situation cannot be addressed here.

Example 12:

Sally has a recipe that needs ¾ teaspoon of butter for every 2 cups of milk. If Sally increases the amount of milk to 3 cups of milk, how many teaspoons of butter are needed?

Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion $\frac{3}{4} = \frac{x}{3}$ to show the relationship between butter and milk.

One possible solution is to recognize that $2 \cdot 1 \frac{1}{2} = 3$ so $\frac{3}{4} \cdot 1 \frac{1}{2} = x$. The amount of butter needed would be 1 teaspoons.

A second way to solve this proportion is to use cross-multiplication $\frac{3}{4} \cdot 3 = 2x$. Solving for $x$ would give 1 teaspoons of butter.

Misconceptions:
1. Students believe all percents must be between 1 and 100%
2. Students believe that a percent that has a decimal in it is already in decimal form (43.5% in decimal form is 43.5)
3. In multiple discount problems, students want to combine the discounts instead of working them sequentially (such as 25% off then 10% off makes students want to use 35% instead of finding 25% off and the 10% off).
### Proportionality:
**TEKS: 7.5**

The student applies mathematical process standards to use geometry to describe or solve problems involving proportional relationships. The student is expected to:

#### 7.5(A) Generalize the critical attributes of similarity, including ratios within and between similar shapes

Student’s understanding includes attention to the identification of common features (attributes), use of examples and non-examples, classifying or grouping items, naming or labeling the group, and using the concept in ongoing activities.

Students will identify corresponding sides and corresponding congruent angles of similar figures using the traditional notation of curved lines for the angles. Students will write proportions to express the relationships between the lengths of corresponding sides of similar figures (relationships between sides of similar figures and within sides of similar figures).

Including, but not limited to:
- Notation for similar figures aligning corresponding angles and sides:
\[ \triangle ABC \sim \triangle LMN \]
- Corresponding parts of similar figures (alignment as indicated in similarity statement)
  - Corresponding angles are congruent:\n    \[
    \angle A \equiv \angle L, \angle B \equiv \angle M, \angle C \equiv \angle N
    \]
  - Corresponding sides are related proportionally:
    \[
    \frac{AB}{LM} = \frac{BC}{MN} = \frac{AC}{LN}
    \]
  - Likewise,
    \[
    \frac{AB}{BC} = \frac{LM}{MN}
    \]

Example Problem/ Tasks

**Example 1:**

Which of the following statements is true for the following set of similar figures?

\[\frac{RP}{RQ} = \frac{XV}{VW}\]

\[\frac{RP}{VW} = \frac{XV}{PQ}\]

\[\frac{PQ}{RQ} = \frac{VW}{XW}\]

\[\frac{RQ}{XW} = \frac{RP}{VX}\]
### Example 2:

Which of the following statements is true?

- A  All trapezoids are similar.
- B  All isosceles triangles are similar.
- C  All equilateral triangles are similar.
- D  All rectangles are similar.

### Misconceptions:
1. Students may believe that corresponding angles are also proportional in similar figures
2. Over generalizing the phrase “same shape, different size”
3. Students believe the scale factor has to be a whole number

### 7.5(B) Describe as the ratio of the circumference of a circle to its diameter

The student will derive an approximation for pi by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. Students may have to determine how to find the radius or diameter when given the circumference, based on the understanding of pi = circ/diam. Students will also be able to leave answers as exact (with pi on the answer) rather than getting an approximation.

**Example:**

Given several circular items (jars, lids, coffee cans, plates, etc) and tape measures, students will measure and record the diameter and circumference of each item. Then they will divide the circumference by the diameter to the nearest tenth to discover the relationship (pi).

### Misconceptions:
1. Pi changes based on the size of the circle
2. Students believe pi = 3.14 (this is an approximation, not the true value)

### 7.5(C) Solve mathematical and real-world problems involving similar shape and scale drawings

Students determine the dimensions of figures when given a scale. Students identify the scale factor given two figures. Using a given scale drawing, students reproduce the drawing at a different scale. Students understand that the lengths will change by a factor equal to the product of the magnitude of the two size transformations.

**Example 1:**

Julie shows the scale drawing of her room below. If each 2 cm on the scale drawing equals 5 ft, what are the actual...
Example 2:

If the rectangle below is enlarged using a scale factor of 1.5, what will be the perimeter and area of the new rectangle?

Solution:
The perimeter is linear or one-dimensional. Multiply the perimeter of the given rectangle (18 in.) by the scale factor (1.5) to give an answer of 27 in. Students could also increase the length and width by the scale factor of 1.5 to get 10.5 in. for the length and 3 in. for the width. The perimeter could be found by adding 10.5 + 10.5 + 3 + 3 to get 27 in. The area is two-dimensional so the scale factor must be squared. The area of the new rectangle would be 14 x 1.52 or 31.5 in2.

Example 3:

In Ponce de Leon Park there is a flagpole that is exactly 50 feet tall. Nearby there is a pine tree that appears to be nearly as tall as the flagpole. On a sunny afternoon the flagpole is casting a 15-foot-long shadow on the ground. The pine tree is casting a 12-foot-long shadow on the ground at the same time.
How much taller than the pine tree is the flagpole?

**Misconceptions:**
1. Students believe the scale factor has to be a whole number
2. Students look for the difference in lengths of corresponding sides, instead of the ratio. They think it is an additive relationship instead of a multiplicative.
3. When solving for the missing length students use 3 and 8 rather than 3 and 11. See picture below.

**Proportionality:**
**TEKS: 7.6**

<table>
<thead>
<tr>
<th>TEKS: 7.6</th>
<th>The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships. The student is expected to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.6(A)</td>
<td>Represent sample spaces for simple and compound events using lists and tree diagrams</td>
</tr>
<tr>
<td>7.6(B)</td>
<td>Select and use different simulations to represent simple and compound events with and without technology</td>
</tr>
</tbody>
</table>

Example 1: (7.6 I)

Jason is tossing a fair coin. He tosses the coin ten times and it lands on heads eight times. If
| 7.6(C) Make predictions and determine solutions using experimental data for simple and compound events | Jason tosses the coin an eleventh time, what is the probability that it will land on heads?

**Example 2:** (7.6 A)

How many ways could the 3 students, Amy, Brenda, and Carla, come in 1st, 2nd and 3rd place? Represent this information using a list and tree diagram.

**Example 3:** (7.6 C)

Conduct an experiment using a Styrofoam cup by tossing the cup and recording how it lands.
- How many trials were conducted?
- How many times did it land right side up?
- How many times did it land upside down?
- How many times did it land on its side?
- Determine the probability for each of the above results

Solution:

Making an organized list will identify that there are 6 ways for the students to win a race
A, B, C
A, C, B
B, C, A
B, A, C
C, A, B
C, B, A

**Example 4:** (7.6 I)

Students conduct a bag pull experiment. A bag contains 5 marbles. There is one red marble, two blue marbles and two purple marbles. Students will draw one marble without replacement and then draw another. What is the sample space for this situation? Explain how the sample space was determined and how it is used to find the probability of drawing one blue marble followed by another blue marble.

**Example 5:** (7.6 D)

A fair coin will be tossed three times. What is the probability that two heads and one tail in any order will results?

Solution:

HHT, HTH and THH so the probability would be \(\frac{3}{8}\).
### Misconceptions:
1. Vocabulary related to probability is frequently used in the “real-world” but not necessarily in the true mathematical meanings, for example, probability and odds are often used interchangeably.

### Expressions, equations, and relationships: TEK: 7.7

The student applies mathematical process standards to represent linear relationships using multiple representations. The student is expected to:

7.7(A) Represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to the form \( y = mx + b \)

Extend and apply their understanding of independent and dependent quantities (from TEK 6.6A&B). Simplify algebraic expressions and equations by combining like terms. Substitute values and evaluate algebraic equations. Identify the rate of change from a table, graph, equation, or verbal description. Given one representation, students need to be able to create any of the others. Make connections between representations of the same linear relationship.

**Example 1:**

The graph below represents the cost of gum packs as a unit rate of $2 dollars for every pack of gum. The unit rate is represented as $2/pack. Represent the relationship using a table and an equation.

![Graph of linear relationship](graph.png)

**Solution:**

**Table:**

<table>
<thead>
<tr>
<th>Number of Packs of Gum ((g))</th>
<th>Cost in Dollars ((d))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Equation: \( d = 2g \), where \( d \) is the cost in dollars and \( g \) is the packs of gum.
Example 2:

Use the given information to fill in the missing information.

Mason pays $2.00 per hour to use a computer and an additional charge of $2.50

<table>
<thead>
<tr>
<th>Number of Hours</th>
<th>Process</th>
<th>Amount Paid ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>$y$</td>
<td></td>
</tr>
</tbody>
</table>

**Equation:**

\[ (g \times 2) = d \]

**Misconceptions:**
1. A common error is to reverse the position of the variables when writing equations. Students may find it useful to use variables specifically related to the quantities rather than using \(x\) and \(y\). Constructing verbal models can also be helpful. A student might describe the situation as “the number of packs of gum times the cost for each pack is the total cost in dollars”. They can use this verbal model to construct the equation. Students can check their equation by substituting values and comparing their results to the table. The checking process helps student revise and recheck their model as necessary. The number of packs of gum times the cost for each pack is the total cost.  

**Expressions, equations, and relationships:**

**TEK: 7.8**

7.8(A) Model the relationship between the volume of a rectangular prism and a rectangular pyramid having both congruent bases and heights and connect that relationship to the formulas

The volume of a pyramid is \(\frac{1}{3}\) the volume of prism with congruent bases and height. Therefore, it takes the volume of 3 pyramids to fill the prism

Exploration 1:
Using hollow geometric solids, have students estimate the relationship between the volume of the pyramid and prism with congruent bases and height. How many pyramids would fit in the prism?
| Fill the pyramid with rice, water, etc and pour into the prism. Ask the students to refine their estimate. How full is the prism? How many pyramids full would fit in the prism? Continue to fill the pyramid and pour into the prism to demonstrate that it takes 3 pyramid-fulls to fill the prism. Therefore, the volume of the pyramid is \( \frac{1}{3} \) the volume of the prism, hence the formula.  

Exploration 2:  
From GATAR and Mathematics TEKS Connections: Geometry  
Students use nets/templates to create three congruent square pyramids and manipulate them to form a prism of the same base area and height.  

**Misconceptions:**  
1. Students often confuse the vocabulary base, length, height, “B” (base area), when moving between 2- and 3- dimensional figures.  
2. Students believe that whatever face the prism is sitting on is the base of the figure.  
3. Students should connect prior learning related to the volume of rectangular prisms and pyramids to triangular prisms and pyramids.  
   For example: The volume of a pyramid is \( \frac{1}{3} \) the volume of prism with congruent bases and height. Therefore, it takes the volume of 3 pyramids to fill the prism.  
   Students should understand that  
   \[ V = \frac{1}{3}Bh, \text{ where } B = \frac{1}{2}bh \text{ (area of base triangle)} \]  

| 7.8(B) Explain verbally and symbolically the relationship between the volume of a triangular prism and a triangular pyramid having both congruent bases and heights and connect that relationship to the formulas  
Students should connect prior learning related to the volume of rectangular prisms and pyramids to triangular prisms and pyramids.  
For example: The volume of a pyramid is \( \frac{1}{3} \) the volume of prism with congruent bases and height. Therefore, it takes the volume of 3 pyramids to fill the prism.  
Students should understand that  
\[ V = \frac{1}{3}Bh, \text{ where } B = \frac{1}{2}bh \text{ (area of base triangle)} \]  

**Misconceptions:**  
Students often confuse the vocabulary base, length, height, “B” (base area), when moving between 2- and 3- dimensional figures.  

| 7.8(C) Use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas  
Understand that circumference is the perimeter of a circle and measures the distance around vs. area representing the space covered.  
   1. Use various tools to measure dimensions of circular figures  
   2. Write algebraic equations/formulas to represent the relationships discovered in an exploration  
   3. Eventually, understand that pi is the ratio of circumference of a circle to its diameter (7.5B)  

**Example Circumference Tasks:**  
Ex.1 - Students measure the diameter of a circular figure with string or pipe cleaner. They then determine how many of those lengths would be needed to go around the entire circle. Students should determine it is close to three. They then create a equation/formula to represent this
Generate a graph by using the diameters and circumferences of round objects. Once the data has been collected and a relationship has been established by analyzing that data, determine a formula for the circumference of a circle.

Example Area Tasks

Example 1:

The illustration shows the relationship between the circumference and area. If a circle is cut into wedges and laid out as shown, a parallelogram results. Half of an end wedge can be moved to the other end a rectangle results. The height of the rectangle is the same as the radius of the circle. The base length is \( \frac{1}{2} \times \text{circumference} (2\pi r) \). The area of the rectangle (and therefore the circle) is found by the following calculations:

\[
\text{Area}_{\text{rect}} = \text{Base} \times \text{Height} \\
\text{Area} = \frac{1}{2} (2\pi r) \times r \\
\text{Area} = \pi r \times r \\
\text{Area} = \pi r^2
\]

Students solve problems (mathematical and real-world) involving circles or semi-circles.

Note: Because \( \pi \) is an irrational number that neither repeats nor terminates, the measurements are approximate when 3.14 is used in place of \( \pi \).

Example Area Task 2:

From GATAR:

Area of a Circle Activity:
Students will use grid paper to derive a general rule for estimating the area of a circle. Students will also participate in a hands-on investigation of the relationship between the formula for computing the area of a circle and the formula for the area of a rectangle. Using this relationship, students will derive the formula for computing
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<thead>
<tr>
<th>Expressions, equations, and relationships:</th>
<th>The student applies mathematical process standards to solve geometric problems. The student is expected to:</th>
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<tbody>
<tr>
<td><strong>TEK: 7.9</strong></td>
<td><strong>7.9(A) Solve problems involving the volume of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids</strong></td>
</tr>
<tr>
<td></td>
<td>“Know the formula” does not mean memorization of the formula. To “know” means to have an understanding of why the formula works and how the formula relates to the measure (area and volume) and the figure. This understanding should be for all students.</td>
</tr>
<tr>
<td></td>
<td><strong>Example:</strong></td>
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<tr>
<td></td>
<td>Jennie purchased a box of crackers from the deli. The box is in the shape of a triangular prism (see diagram below). If the volume of the box is 3,240 cubic centimeters, what is the height of the triangular face of the box? How much packaging material was used to construct the cracker box? Explain how you got your answer.</td>
</tr>
<tr>
<td></td>
<td><strong>Solution:</strong></td>
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<tr>
<td></td>
<td>Volume can be calculated by multiplying the area of the base (triangle) by the height of the prism. Substitute given values and solve for the area of the triangle</td>
</tr>
</tbody>
</table>

\[ V = Bh \]

\[ 3,240 \text{ cm}^3 = B \times (30\text{cm}) \]

\[ 3,240 \text{ cm}^3 = B(30\text{cm}) \]

\[ 30 \text{ cm} \times 30 \text{ cm} \]

\[ 108 \text{ cm}^2 = B \text{ (area of the triangle)} \]
To find the height of the triangle, use the area formula for the triangle, substituting the known values in the formula and solving for height. The height of the triangle is 12 cm. The problem also asks for the surface area of the package. Find the area of each face and add:

2 triangular bases: \( \frac{1}{2} (18 \text{ cm})(12 \text{ cm}) = 108 \text{ cm}^2 \times 2 = 216 \text{ cm}^2 \)
2 rectangular faces: \( 15 \text{ cm} \times 30 \text{ cm} = 450 \text{ cm}^2 \times 2 = 900 \text{ cm}^2 \)
1 rectangular face: \( 18 \text{ cm} \times 30 \text{ cm} = 540 \text{ cm}^2 \)
Adding \( 216 \text{ cm}^2 + 900 \text{ cm}^2 + 540 \text{ cm}^2 \) gives a total surface area of \( 1656 \text{ cm}^2 \).

**Misconceptions:**
1. Students often assume that the face a solid figure is sitting on is its base, this causes particular difficulty in triangular prisms.
2. Students often confuse the vocabulary base, length, height, “B” (base area), when moving between 2- and 3-dimensional figures.

<table>
<thead>
<tr>
<th>7.9(B) Determine the circumference and area of circles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find the circumference of a circle by substituting a value for the diameter or the radius into the formula ( C=\pi d \text{ or } C=2\pi r )</td>
</tr>
<tr>
<td>Understand the relationship between circumference, diameter, and radius. Understand that pi is a ratio of circumference to diameter. Student should be able to solve for any of these when given another. Students may be asked to determine the radius or diameter when given the circumference or area. Therefore they will need to be able to solve equations for which the variable is not already isolated.</td>
</tr>
<tr>
<td>( 113.04 = \pi d \text{ } vs \text{ } \pi(6) = C )</td>
</tr>
<tr>
<td>Example 1: The seventh grade class is building a mini-golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 ft in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might someone communicate this information to the salesperson to make sure he receives a piece of carpet that is the correct size? Use 3.14 for ( \pi ).</td>
</tr>
<tr>
<td>Solution:</td>
</tr>
<tr>
<td>Area =</td>
</tr>
<tr>
<td>Area =</td>
</tr>
<tr>
<td>Area = ( 78.5 )</td>
</tr>
<tr>
<td>To communicate this information, ask for a 9ft by 9ft square of carpet.</td>
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<tr>
<td>Example 2: The center of the circle is at ((5,-5)). What is the area of the circle?</td>
</tr>
<tr>
<td>Solution:</td>
</tr>
<tr>
<td>The radius of the circle is 4. Using the formula, Area = the area of the circle is approximately 50.24.</td>
</tr>
<tr>
<td>Students build on their understanding of area from 6th grade to find the area of left-over materials</td>
</tr>
</tbody>
</table>
when circles are cut from squares and triangles or when squares and triangles are cut from circles.

**Misconceptions:**
1. Students may confuse the terms circumference and area with what they represent in a circle.
2. Common computation errors include: multiplying by 2 instead of squaring the radius, using radius in place of diameter (and vice versa), not following the order of operations when solving,

<table>
<thead>
<tr>
<th>7.9(C) Determine the area of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, triangles, semi-circles, and quarter circles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students need to be able to substitute values into a formula and evaluate. Students need to be able to break a composite figure into shapes for which there is a known area formula. Students will use individual formulas learned previously in conjunction to find the area of composite figures. Students should understand that there is often more than one way that a composite figure can be broken up.</td>
</tr>
</tbody>
</table>

**Example Problems/ Tasks:**

**Example 1:**
If a circle is cut from a square piece of plywood, how much plywood would be left over?

![Diagram of a circle cut from a square]

**Solution:**
The area of the square is 28x28 or 784. The diameter of the circle is equal to the length of the side of the square, or 28", so the radius would be 14". The area of the circle would be approximately 615.44. The difference in the amounts (plywood left over) would be 168.56 (784-615.44).
Example 2:
What is the perimeter of the inside of the track?

Solution: The ends of the track are two semicircles, which would form one circle with a diameter of 62m. The circumference of this part would be 194.68m. Add this to the two lengths of the rectangle and the perimeter is 2194.68m.

Example 3:
The sixth grade class at Hernandez School is building a giant wooden H for their school. The “H” will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.

1. How large will the H be if measured in square feet?
2. The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?

Solution:

1. One solution is to recognize that, if filled in, the area would be 10 feet tall and 10 feet wide or 100 feet squared. The size of one piece removed is 5 feet by 3.75 feet or 18.75 feet squared. There are two of these pieces. The area of the “H” would be 100 feet squared - 18.75 feet squared - 18.75 feet squared, which is 62.5 feet squared. A second solution would be to decompose the “H” into two tall rectangles measuring 10ft by 2.5ft and one smaller rectangle measuring 2.5ft by 5ft. The area of each tall rectangle would be 25 feet squared and the area of the smaller rectangle would be 12.5 feet squared. Therefore the area of the “H” would be 25 feet squared + 25 feet squared + 12.5 feet squared or 62.5 feet squared.

2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5ft by 5ft. Cut two pieces of wood in half to create four pieces 5 ft by 2.5ft. These pieces will make the two taller rectangles. A third piece would be cut to measure 5 ft by 2.5ft to create the middle space.
### Misconceptions:
1. Students think that there are formulas that work for all shapes and not understand that they may need to use multiple formulas together for composite shapes.
2. Students may have trouble using the given measurements to find missing dimensions that are needed to solve for the composite area.

<table>
<thead>
<tr>
<th>7.9(D)</th>
<th>Solve problems involving the lateral and total surface area of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid by determining the area of the shape’s net.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students should understand that surface area is the sum of the area of faces. Total surface area is the sum of all faces including the bases. Lateral surface area is the sum of the faces excluding the bases. “Knowing the formula” does not mean memorization of the formula. To ‘know” means to have an understanding of why the formula works and how the formula relates to the measure (area and volume) and the figure. This understanding should be for all students. Surface area formulas are not the expectation with this standard. Building on work with nets in the 6th grade, students should recognize that finding the area of each face of a three-dimensional figure and adding the areas will give the surface area. No nets will be given at this level; however, students could create nets to aid in surface area calculations.</td>
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</table>

**Example 1:**

The surface area of a cube is 96 in$^2$. What is the volume of the cube?

**Solution:**

The area of each face of the cube is equal. Dividing 96 by 6 gives an area of 16 in$^2$ for each face. Because each face is a square, the length of the edge would be 4 in. The volume could then be found by multiplying $4 \times 4 \times 4 = 64$ in$^3$.

**Example 2:**

Huong covered the box to the right with sticky-backed decorating paper. The paper costs 3¢ per square inch. How much money will Huong need to spend on paper?

**Solution:**

The surface area can be found by using the dimensions of each face to find the area and multiplying by 2:

- **Front:** $7 \text{ in.} \times 9 \text{ in.} = 63 \text{ in}^2 \times 2 = 126 \text{ in}^2$
- **Top:** $3 \text{ in.} \times 7 \text{ in.} = 21 \text{ in}^2 \times 2 = 42 \text{ in}^2$
- **Side:** $3 \text{ in.} \times 9 \text{ in.} = 27 \text{ in}^2 \times 2 = 54 \text{ in}^2$

![Diagram of a rectangular prism]
The surface area is the sum of these areas, or 222 in². If each square inch of paper cost $0.03, the cost would be $6.66.

Example 3:

Jennie purchased a box of crackers from the deli. The box is in the shape of a triangular prism (see diagram below). If the volume of the box is 3,240 cubic centimeters, what is the height of the triangular face of the box? How much packaging material was used to construct the cracker box? Explain how you got your answer.

![Diagram of a triangular prism]

**Solution:**

Volume can be calculated by multiplying the area of the base (triangle) by the height of the prism. Substitute given values and solve for the area of the triangle

\[ V = Bh \]

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Adding 216 cm² + 900 cm² + 540 cm² gives a total surface area of 1656 cm².
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<th>Expressions, equations, and relationships:</th>
<th>The student applies mathematical process standards to use one-variable equations and inequalities to represent situations. The student is expected to:</th>
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<tr>
<td>TEK: 7.10</td>
<td></td>
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<tr>
<td>7.10(A) Write one-variable, two-step equations and inequalities to represent constraints or conditions within problems</td>
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<tr>
<td>7.10(B) Represent solutions for one-variable, two-step equations and inequalities on number lines</td>
<td>Identify unknowns from a problem situation and use variables to represent those unknowns. Identify and use information given in a situation to represent constants and coefficients in an equation or inequality. Choose appropriate operations to represent a situation. Determine whether a situation is an equation or inequality. Draw and label a number line with values appropriate for the problem situation. Plot single points on a number line. Understand which inequality symbol includes the solution number (whether to color it in or not). Know which way to shade the number line for the given inequality symbol. Read an inequality solution whether the variable is first or last in the inequality statement. Understand how operations in an equation or inequality behave in the real world. Know that the variable is the piece of information that needs to be solved for in the problem situation.</td>
</tr>
<tr>
<td>7.10(C) Write a corresponding real-world problem given a one-variable, two-step equation or inequality</td>
<td>Example 1: Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 and spend the rest on t-shirts. Each t-shirt costs $8. Write an inequality for the number of t-shirts she can purchase.</td>
</tr>
</tbody>
</table>
|                                           | Solution: x-cost of one t-shirt

\[ X = 4.75 \] is the most t-shirts she can purchase |
<p>|                                           | Example 2: Steven has $25 to spend. He spend $10.81, including tax, to buy a new DVD. He needs to save $10 but he wants to buy a snack. If peanuts cost $0.38 per package including tax, what is the maximum number of packages that Steven can buy? |
|                                           | Solution: |</p>
<table>
<thead>
<tr>
<th>X=number of packages of peanuts</th>
<th>X = 11.03  Steven can buy 11 packages of peanuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Misconceptions:</td>
<td></td>
</tr>
<tr>
<td>1. Students think the equation has to represent how they would solve the problem, not necessarily what the problem describes as the situation. (writing the equation in x= form, instead of with the x embedded)</td>
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<tr>
<td>2. Students assume that the direction the inequality symbol is pointing is always the direction they shade on the number line.</td>
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<tr>
<td>3. Students think that a number line always have to be drawn starting with 0 or 1.</td>
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<table>
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<tr>
<th>Expressions, equations, and relationships: TEK: 7.11</th>
<th>The student applies mathematical process standards to use statistical representations to analyze data. The student is expected to:</th>
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<tr>
<td>7.11(A)  Model and solve one-variable, two-step equations and inequalities</td>
<td>Students will draw pictures to represent an equation/inequality and its constants and variables. Students will understand that an equation/inequality has to stay balanced during the solving process. Students will relate the actions taken to solve a model to the inverse operations used to solve without models. Students need to know the inverse operation for all operations. Students need to understand that you perform the order of operations in reverse when solving an equation/inequality. Students need to know that when solving an inequality, if you multiply or divide by a negative number you must flip the inequality symbol.</td>
</tr>
<tr>
<td>Example 1:</td>
<td></td>
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<tr>
<td>500 – 25w &gt; 200</td>
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<tr>
<td>500 – 500 – 25w &gt; 200 – 500</td>
<td></td>
</tr>
<tr>
<td>Subtract 500 from both sides.</td>
<td></td>
</tr>
<tr>
<td>-25w &gt; -300</td>
<td></td>
</tr>
<tr>
<td>-25</td>
<td></td>
</tr>
<tr>
<td>Divide by -25</td>
<td></td>
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<tr>
<td>w ≤ 12</td>
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<tr>
<td>Reverse your sign since you divided by a negative number.</td>
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</tbody>
</table>
Example 2:
The youth group is going on a trip to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.

Solution:
x = cost of one pass

\[
\begin{array}{c|c|c}
& x & x & 11 \\
\hline
2x + 11 & = & 52 \\
2x & = & 41 \\
x & = & 20.50
\end{array}
\]

Example 3:
Solve: \( \frac{2}{3}x - 4 = -16 \)

Solution:
\[
\begin{align*}
\frac{2}{3}x - 4 &= -16 \\
\frac{2}{3}x &= -12 \\
\frac{2}{3}x + 4 &= -12 + 4 \\
\frac{2}{3}x &= -8 \\
x &= -12 \\
\frac{2}{3}x &= -18 \\
x &= -12 \times \frac{3}{2} \\
x &= -18
\end{align*}
\]

Students could also reason that if \( \frac{2}{3} \) of some amount is -12 then \( \frac{1}{3} \) is -6. Therefore, the whole amount must be 3 times -6 or -18.

Misconceptions:
1. Students think that solving equations/inequalities also follows the order of operations
2. Students think answers must always come out to a whole number
3. Students think that the variable always needs to be on the left side of the equal or inequality symbol in a solution
7.11(B) Determine if the given value(s) make(s) one-variable, two-step equations and inequalities true

The student will be able to substitute values for variables in equations/inequalities. Students will evaluate an equation/inequality using order of operations. Students understand that equations have a single solution and inequalities have a set of solutions.

Example:

Check that the given number is a solution to the corresponding inequality.

a) \( a = 10; \ 20a \leq 250 \)

b) \( b = -0.5; \ \frac{3-b}{5} > -4 \)

c) \( x = \frac{3}{4}; \ 4x + 5 \leq 8 \)

Solution

Replace the variable in each inequality with the given value.

a) \[ 20(10) \leq 250 \]
\[ 200 \leq 250 \]

This statement is true. This means that \( a = 10 \) is a solution to the inequality \( 20a \leq 250 \).

Note that \( a = 10 \) is not the only solution to this inequality. If we divide both sides of the inequality by 20, we can write it as \( a \leq 12.5 \). This means that any number less than or equal to 12.5 is also a solution to the inequality.

Example I: Find the solution set of \( 11 - 5w = 1 \) from the replacement set \( \{0, 2, 4\} \).

1. \[ 11 - 5(0) = 1 \ ? \]
   
   \[ 11 - 0 = 1 \ ? \]
   
   \[ 11 = 1 \ ? \text{ No. Not a solution.} \]

2. \[ 11 - 5(2) = 1 \ ? \]
   
   \[ 11 - 10 = 1 \ ? \]
   
   \[ 1 = 1 \ ? \text{ Yes. Solution.} \]

3. \[ 11 - 5(4) = 1 \ ? \]
   
   \[ 11 - 20 = 1 \ ? \]
   
   \[ -9 = 1 \ ? \text{ No. Not a solution.} \]

Thus, the solution set is \( \{2\} \).
| 7.11(C) Write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships | Students need to be able to recall previous geometry learning including angles and angle sums (could also include sum of angles in other polygons, angle pairs, perimeter, area, volume). Students identify given information in a geometric figure and relate to a formula. Substitute information from a geometric figure into a formula and create an equation to represent a geometric situation/relationship.  

Example: Find the measure of $\angle Q$ below.  

![Diagram of angles](image)  

Again, we need to find how the known angle and the unknown angle are related. This time angle $Q$ is not opposite the known angle. It is adjacent, because together they form a straight line. What do we know about adjacent angles? **They add up** to $180^\circ$. Therefore we can use the measure of the known angle to solve for angle $Q$.  

\[138^\circ + \angle Q = 180\]  
\[\angle Q = 180 - 138\]  
\[\angle Q = 42^\circ\]  

**Angle $Q$ must be** $42^\circ$. |
Example 1:
Write and solve an equation to find the measure of angle $x$.

Solution:
Find the measure of the missing angle inside the triangle ($180 - 90 - 40$), or $50^\circ$.
The measure of angle $x$ is supplementary to $50^\circ$, so subtract 50 from 180 to get a measure of $130^\circ$ for $x$.

Example 2:
Find the measure of angle $x$.

Solution:
First, find the missing angle measure of the bottom triangle ($180 - 30 - 30 = 120$). Since the 120 is a vertical angle to $x$, the measure of $x$ is also $120^\circ$.

Example 3:
Find the measure of angle $b$.

Solution:
Because, the $45^\circ$, $50^\circ$ angles and $b$ form are supplementary angles, the measure of angle $b$ would be $85^\circ$. The measures of the angles of a triangle equal $180^\circ$ so $75^\circ + 85^\circ + a = 180^\circ$. The measure of angle $a$ would be $20^\circ$.

Example 4:
A rectangle is twice as long as its width. One way to write an expression to find the perimeter would be \( w + w + 2w + 2w \). Write the expression in two other ways.

**Solution:**
6\(w\) or 2(2\(w\))

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**Example 6:**
An equilateral triangle has a perimeter of \(6x + 15\). What is the length of each side of the triangle?

**Solution:**
\(3(2x + 5)\), therefore each side is \(2x + 5\) units long.

**Misconceptions:**
1. Students may not understand that equations can be used to represent geometric situations.
2. Students may not understand that formulas are just a kind of equation.

<table>
<thead>
<tr>
<th><strong>Measurement and data:</strong></th>
<th><strong>The student applies mathematical process standards to use statistical representations to analyze data. The student is expected to:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TEK: 7.12</strong></td>
<td><strong>7.12(A) Compare two groups of numeric data using comparative dot plots or box plots by comparing their shapes, centers, and spreads</strong></td>
</tr>
<tr>
<td></td>
<td>Students will interpret box plots and dot plots individually. Students understand what components of box plots mean (median, extremes, outliers, quartiles) in relation to the situation. Students understand what the shape of a box or dot plot implies about a situation (likewise with center and spread).</td>
</tr>
<tr>
<td></td>
<td><strong>Examples:</strong></td>
</tr>
</tbody>
</table>
Describe the least and greatest amounts of tickets purchased. The smallest quantity of tickets purchased is 225. The greatest number of tickets purchased is 425.

Describe the range in attendance for the plays. The range between the greatest and least number of attendees is 205. The least number of attendees is 200. The greatest number of attendees is 405.

Compare the data for number of tickets purchased with the data for actual attendance. It is evident that not all tickets purchased were used to attend the shows. The median number of tickets purchased is 350. The median number of attendance is 325. The majority of the data values fall in the lower quartile for both sets of data, purchases and attendance.

You can see how useful box-and-whisker plots are for analyzing and interpreting data.
**CDs Owned: Boys vs. Girls**

Compare the median number of CDs owned by boys and girls. The median number of CDs owned by boys is four more than the girls. The median number of CDs owned by boys is 18. The median number of CDs owned by girls is 14.

What is the least number of CDs owned by boys and girls? The smallest amount of CDs owned by both boys and girls is 3.

Identify the greatest number of CDs owned by both girls and boys. The greatest amount of CDs owned by girls is 37. The greatest amount of CDs owned by boys is 31.

What conclusions can be drawn about the number of CDs owned by girls? You can see that more than half of the data falls in the upper quartile. Therefore, more than half of the girls own more than the median number of CDs.

What conclusions can be drawn about the number of CDs owned by boys? You can see that more than half of the data falls in the lower quartile. Therefore, more than half of the boys own less than the median number of CDs.
### Example:

Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the idea scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5.

![6-Trait Writing Rubric](image)

### Misconceptions:

1. Students think that a larger quartile box means there is more data in it.
2. Students assume that a “x” or “dot” on a dot plot always represents 1.

<table>
<thead>
<tr>
<th>7.12(B) Use data from a random sample to make inferences about a population</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.12(C) Compare two populations based on data in random samples from these populations, including informal comparative inferences about differences between the two populations</td>
</tr>
</tbody>
</table>

Students need to apply proportional reasoning to make predictions about a population. Students extend information from samples of populations to make generalizations about the whole populations. Students compare data from samples and interpret their implications about the differences of the whole populations. Students need to be able to identify whether a sample is random or not.
Example 1:
The two data sets below depict random samples of the management salaries in two companies. Based on the salaries below which measure of center will provide the most accurate estimation of the salaries for each company?

- Company A: 1.2 million, 242,000, 265,500, 140,000, 281,000, 265,000, 211,000
- Company B: 5 million, 154,000, 250,000, 250,000, 200,000, 160,000, 190,000

Solution:
The median would be the most accurate measure since both companies have one value in the million that is far from the other values and would affect the mean.

Example 1:
Below is the data collected from two random samples of 100 students regarding student’s school lunch preference. Make at least two inferences based on the results.

<table>
<thead>
<tr>
<th>Student Sample</th>
<th>Hamburger</th>
<th>Tacos</th>
<th>Pizza</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>12</td>
<td>14</td>
<td>74</td>
<td>100</td>
</tr>
<tr>
<td>#2</td>
<td>12</td>
<td>11</td>
<td>77</td>
<td>100</td>
</tr>
</tbody>
</table>

Solution:
- Most students prefer pizza.
- More people prefer pizza and hamburgers and tacos combined.

For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Misconceptions:
1. Students oversimplify how to randomly sample. (They assume that sampling every 5th person, makes it random.)
2. Students believe that the characteristics of the sample match exactly the characteristics of the population.
<table>
<thead>
<tr>
<th>Personal financial literacy: TEK: 7.13</th>
<th>The student applies mathematical process standards to develop an economic way of thinking and problem solving useful in one’s life as a knowledgeable consumer and investor. The student is expected to:</th>
</tr>
</thead>
</table>

| 7.13(A) Calculate the sales tax for a given purchase and calculate income tax for earned wages | Students should understand how to find percent of a number. Students need to understand what the word tax stands for. Students need to understand the difference between sales tax and income tax. Students should be able to find the final cost of an item after tax is added. |

**Example:**

refer to relevant examples in 7.4D


**Misconceptions:**

1. Students are drawn to answer choices that comprise the total with tax. They make errors in understanding what the problem is asking.
2. Students can get confused when converting percents to decimals and should be taught to check their answer for reasonableness to further understand percent of a number.

**Sample Lessons:**

http://smartertexas.org/?page_id=914

| 7.13(B) Identify the components of a personal budget, income; planned savings for college, retirement, and emergencies; taxes; and fixed and variable expenses, and calculate what percentage each category comprises of the total budget | Student need to be able to calculate the percent of a number (for each category of a budget) or find the percent from the part and whole. Understand parts of a budget (ie. income vs expenses). Understand vocabulary related to budgets: income, expenses, assets, liabilities, net worth, wages, taxes, savings). |

**Examples:**

See clarifying tasks from TCEE/SmarterTexas

Sample family budget estimator: http://www.finaid.org/calculators/budget.phtml

Sample lesson on assets and liabilities: www.bused.org/lessons/institute/AccountingBasics.doc

| 7.13(C) Create and organize a financial assets and liabilities record and construct a net worth statement | |
| 7.13(D) Use a family budget estimator to determine the minimum household budget and average hourly wage needed for a family to meet its basic needs in the student’s city or | |
| another large city nearby | **Misconceptions:**  
1. Students may think that everyone needs to budget their money in the same way.  
2. Students may think that people’s expenses are the same, regardless of their career.  
3. Students may think that a budget is a fixed set of values that does not vary. They may not understand that it may vary from month to month.  

Sample Lessons:  
http://smartertexas.org/?page_id=914 |
|---|---|
| 7.13(E) Calculate and compare simple interest and compound interest earnings | Students solve for percent of a number, substitute values into an equation and evaluate, and understand the difference between simple and compound interest and their applications.  

**Examples:**  

**(Simple Interest)**  
$4500.00$ at $9.5\%$ for $6$ years  
$I = Prt$  
$I = (4500.00 \times 0.095 \times 6)$  
$I = 2565.00$  

**(Compound Interest example)**  
Let’s say that I have $1000.00 to invest for $3$ years at a rate of $5\%$ compound interest.  

$M = 1000 \times (1 + 0.05)^3 = 1157.62$.  

You can see that my $1000.00 is worth $1157.62.  


**Misconceptions:**  
1. Students think that simple interest and compound interest can be used interchangeably.  
2. Students may mistake the amount of interest for the total, instead of adding it to the principle.  

Sample Lessons:  
http://smartertexas.org/?page_id=914 |
| 7.13(F) Analyze and compare monetary incentives, including sales, rebates, and coupons | Students need to understand the difference between sales, rebates, and coupons. Students need to be able to calculate the price of an item after a discount, rebate, or coupon. Students need to be able to select the most appropriate incentive for their situation. **Examples:**

Present students with an item to purchase with multiple stores offering the same item with various incentives. Have students determine how much the item would cost at the various stores, determine where they would purchase their items, and justify that selection.

**Misconceptions:**
1. Students think that the larger sized item is always the better buy.
2. Students think that if you get an additional 20% off of something that was already marked down 30%, that it is now 50% off.

**Sample Lessons:**
[http://smartertexas.org/?page_id=914](http://smartertexas.org/?page_id=914) |